Black holes and a scalar field in expanding universe

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Abstract

We consider a model of inhomogeneous universe with the presence of a massless scalar field, where the inhomogeneity is assumed to consist of many black holes. Such model can be constructed by following Lindquist and Wheeler, which has already been investigated without the presence of scalar field to show that an averaged scale factor coincides with that of Friedmann model. In this paper we construct the inhomogeneous universe with an massless scalar field, where it is assumed that the averaged scale factor and scalar field are given by those of the Friedmann model with the presence of the scalar field. All of our calculations are carried out in the frame of Brans-Dicke gravity. In constructing the model of inhomogeneous universe, we define a mass of black hole in Brans-Dicke expanding universe which is equivalent to ADM mass in the epoch of the adiabatic time evolution of the mass, and obtain an equation relating our mass with the averaged scalar field and scale factor. It is not a priori obvious whether or not our mass depends on time during cosmological time evolution. As the results we find that the mass has adiabatic time dependence in a sufficiently late stage of the expansion of the universe, and that its time dependence is qualitatively diffenrent according to the sign of the curvature of the universe: the mass increases decelerately in closed universe case, is constant in flat case, and decreases decelerately in open case. It is also noticed that the mass in the Einstein frame depends on time. Our results that the mass has time dependence should be retained even in the general scalar-tensor gravities and with a potential of the scalar field.

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1 Introduction

Recently strongly motivated by the supernovae observations [1], it becomes common agreement that our universe is going to turn to accelerated expansion in the context of Friedmann universe model. As one possible model, a scalar field can be introduced on Friedmann universe [2]. Such a scalar field plays the role as the quintessence of accelerated expansion. Investigations of scalar field on expanding universe is one of the important current issues.

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On the other hand, our universe consists of the celestial objects like galaxies, stars and possibly black holes. In order to draw a more precise picture of our universe than that of the Friedmann model, it is useful to investigate the effects of the inhomogeneities on the expansion of the universe. So far a few work relating with this issue have been done numerically or in a toy model [3]. In the reference [4], the expansion law in a model of inhomogeneous universe including many black holes has been investigated to result in the agreement of an averaged scale factor with that of the dust-dominated Friedmann model. We call this model the *cell lattice universe*. It is an averaged space time of inhomogeneous universe consisting of many black holes, which is constructed as follows: with considering a regularly tessellated homogeneous and isotropic universe, each cell of the tessellation is replaced by a spherically symmetric black hole. The cell lattice universe can be considered to be the well-defined averaged model of inhomogeneous universe reproducing the expansion law of Friedmann model.

In this paper, we consider the cell lattice universe as the model of an inhomogeneous universe including a massless scalar field. Motivated by the previous paragraph, we assume the averaged scalar field and scale factor are given by those of Friedmann model with the presence of the scalar field. With this assumption we are interested in the question: how should the black hole be affected by the expansion of the universe and the scalar field in order to retain the consistency of the cell lattice universe with the Friedmann model. It is well known that the system of Einstein gravity with scalar fields can be transformed to the frame of scalar-tensor gravity. We consider all of the calculations in this paper with the frame of scalar-tensor gravity, especially of Brans-Dicke gravity for simplicity. In constructing the cell lattice universe in Brans-Dicke gravity, we define a mass of the black hole which is equivalent to the ADM mass of a Schwarzschild black hole in a sufficiently late stage of the expansion of the universe. It is not a priori obvious that our mass depends on time or not. The junction condition in replacing a cell by a black hole gives us an equation relating the mass with the averaged scalar field and scale factor. Through this equation we can investigate whether the mass evolves with time or not, which is an effect of the expansion of the universe on black holes.

In section 2 and 3, Friedmann universe in Brans-Dicke gravity and the method of constructing the cell lattice universe are reviewed respectively. Section 4 is devoted to construction of the cell lattice universe in Brans-Dicke gravity and to analyses of the mass defined also in the section 4. Finally we give summary and discussion in section 5. Throughout this paper, we set $c = 1$.

2 Friedmann Universe in Brans-Dicke Gravity

The action of Brans-Dicke gravity is

$$S = \int d^4x\sqrt{-g}\left[\varphi R - \frac{\omega}{\varphi}(\nabla\varphi)^2 + \mathcal{L}_m\right], \tag{1}$$

where ω is the constant parameter of this theory, φ is the scalar field coupling to gravity and $\mathcal{L}_m$ is the matter Lagrangian which does not include φ. The effective Newton "constant", G_{eff}, is related with the scalar field as $\varphi = (16\pi G_{eff})^{-1}$. The field equations derived from this action are

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= \frac{\omega}{\varphi^2}\left[(\nabla_\mu\varphi)(\nabla_\nu\varphi) - \frac{1}{2}g_{\mu\nu}(\nabla\varphi)^2\right] - \frac{1}{\varphi}\left[g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right]\varphi + \frac{1}{2\varphi}T_{\mu\nu}, \\ \Box\varphi &= \frac{1}{4\omega+6}T^\mu_\mu, \end{aligned} \tag{2}$$

where $T_{\mu\nu}$ is the energy-momentum tensor derived from $\mathcal{L}_m$, which is automatically divergenceless: $\nabla_\mu T^{\mu\nu} = 0$.

We consider the Friedmann universe in Brans-Dicke gravity. The metric is spatially homogeneous and isotropic,

$$ds^2 = -dt^2 + \frac{dr^2}{1 - k\,(r/a(t))^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,, \tag{3}$$

where $k = -1{,}0{,}1$ and $a(t)$ is the scale factor. The matter in the universe is of perfect fluid type,

$$T_{\mu\nu} = \epsilon\, u_\mu u_\nu + p\,(g_{\mu\nu} + u_\mu u_\nu)\,, \tag{4}$$

where u_μ is the 4-velocity of comoving observer, ϵ is the energy density and p is the pressure. Then the field eqs.(2) give three independent equations,

$$\begin{aligned} H^2 + \frac{k}{a^2} &= \frac{\epsilon}{6\,\varphi} - H\frac{\dot\varphi}{\varphi} + \frac{\omega}{6}\left(\frac{\dot\varphi}{\varphi}\right)^2\,, & (5)\\ \ddot\varphi + 3H\dot\varphi &= \frac{1}{4\omega+6}\,(\epsilon - 3p)\,, & (6)\\ \dot\epsilon &= -3H\,(\epsilon + p)\,. & (7) \end{aligned}$$

where $H = \dot a/a$ is the Hubble parameter. This system can be solved provided an equation of state is specified.

For dust-dominated universe the equation of state is $p = 0$, with which the eq.(7) gives $\epsilon = \epsilon_0/a^3$, where ϵ_0 is an integration constant. Then we obtain from eq.(6)

$$\varphi = \frac{\epsilon_0}{4\omega+6}\int^t dt\frac{t+t_0}{a^3}\,, \tag{8}$$

where t_0 is an integration constant. Eqs.(5) and (8) determine the time evolution of the scale factor and the scalar field of dust-dominated Friedmann universe in Brans-Dicke gravity.

For radiation-dominated universe the equation of state is $p = \epsilon/3$, then eq.(7) gives $\epsilon = \epsilon_0/a^4$, where ϵ_0 is an integration constant. By eq.(6) we obtain

$$\varphi = \int^t dt\frac{q}{a^3}\,, \tag{9}$$

where q is an integration constant. Eqs.(5) and (9) determine the time evolution of the scale factor and the scalar field of radiation-dominated Friedmann universe in Brans-Dicke gravity.

For the use of section 4, let us review the frame transformation. By suitable conformal transformation of metric $g_{\mu\nu}$ to the other one $\tilde g_{\mu\nu}$, the action S of eq.(1) can be treated as the Einstein gravity with massless scalar field which does not couple to gravity. Such a transformation is given by

$$g_{\mu\nu} = \exp\left[-\frac{\sigma}{2\omega+3}\right]\tilde g_{\mu\nu}\,, \tag{10}$$

where σ is a scalar field defined by $\varphi = (16\pi G_0)^{-1}\exp[\,\sigma/(2\omega+3)\,]$. Here G_0 is the ordinary Newton constant in the Einstein gravity. Then the action S becomes

$$S = \int d^4x\sqrt{-\tilde g}\left[\frac{1}{16\pi G_0}\left(\tilde R - (\tilde\nabla\sigma)^2\right) + e^{-2\sigma/(2\omega+3)}\mathcal{L}_m\right]. \tag{11}$$

The action expressed by $\tilde{g}_{\mu\nu}$ is said to be in the Einstein frame, while the action of eq.(1) in the original frame. We add tilde to quantities in the Einstein frame. It is obvious by this frame transformation that the Brans-Dicke gravity in the original frame becomes the Einstein gravity in the limit $\omega \to \infty$, that is, $g_{\mu\nu} = \tilde{g}_{\mu\nu}$.

It has already known analytically that, in dust-dominated and flat universe case, $p = 0$ and $k = 0$, only the choice that $t_0 = 0$ lets the scale factor $a(t)$ in Brans-Dicke gravity go over smoothly to that of Einstein gravity in the limit $\omega \to \infty$ [5]. With $t_0 = 0$ we can find the relation

$$\frac{8\omega + 12}{3\omega + 4} = \frac{\epsilon_0\, t^2}{\varphi\, a^3}\,. \tag{12}$$

The scale factor and scalar field in this case can be easily obtained to be

$$\begin{aligned} a(t) &= a_0\, t^{(2\omega+2)/(3\omega+4)}\,, \\ \varphi(t) &= \varphi_0\, t^{2/(3\omega+4)}\,, \end{aligned} \tag{13}$$

where a_0 and φ_0 are constants. In the other curvature cases $k = \pm 1$, the eq.(5) indicates that the universe in early time is dominated by the matter term $\epsilon/(6\varphi)$ ($\propto a^{-3}$), not by the curvature term k/a^2. That is, the curved universe is effectively flat in such the epoch.

3 Cell Lattice Universe

3.1 Strategy to construct the cell lattice universe

Construction of the cell lattice universe is the purely geometric problem, which expresses an averaged inhomogeneous expanding universe including many spherically symmetric objects [4]. The procedure of the construction consists of two steps as follows: *1st step*, regularly tessellate the spherical, flat or hyperbolic spatial section, Σ_t, of homogeneous and isotropic universe by regular polyhedron [6], *2nd step*, replace each cell made of polyhedron by a metric of the spherically symmetric object. With considering the radially directional differential of any great circle's circumference on the two dimensional spherical junction surface in Σ_t, the junction condition in the 2nd step is to connect such a differential in the homogeneous and isotropic universe with that in the metric of a spherically symmetric object. By construction, there are some regions where spherically symmetric metrics replacing the cells overlap, and other regions where the metric do not cover. That is, the cell lattice universe is an averaged space time in such a sense. We consider the scale factor on the junction surface is the averaged scale factor of the cell lattice universe. The averaged cosmological time is set by the proper time of the observer staying on the junction surface.

3.2 Construction of the cell lattice universe

The metric of homogeneous and isotropic universe can be given by the same form as eq.(3). The 1st step to construct the cell lattice universe determines the radius of each spherical cell measured in this metric [6] by requiring that the volume of a spherical cell coincides with that of a regular polyhedron. We denote the radius of the spherical cell by r_c. Since the comoving number density of cells is constant, the cell expands comovingly. Therefore the averaged cosmological time is identical to the time coordinate of the metric (3), t_c, on the junction surface $r = r_c$. The radius r_c depends on t_c.

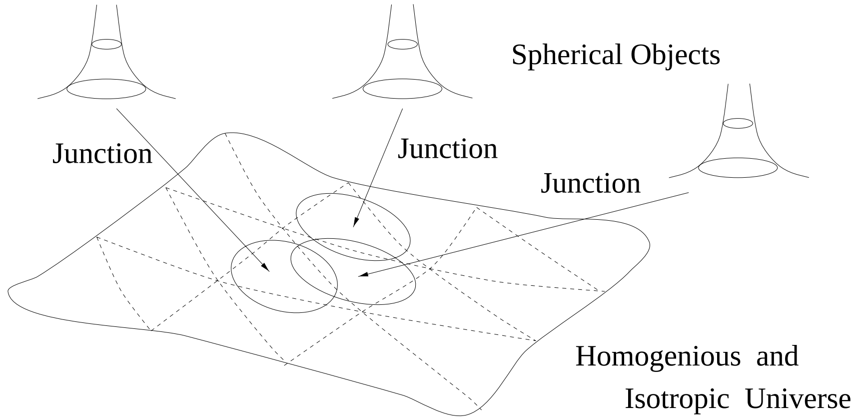

Figure 1: Graphical image of the cell lattice universe. The region surrounded by dashed lines is the regular polyhedron. The circle of real line is the spherical cell. Each cell on the homogeneous and isotropic universe is replaced by a spherically symmetric object. There are some region where some objects overlap while the other regions are covered by no objects.

In proceeding to the 2nd step, we define precisely the "radial direction" in the differential of great circle on the junction surface. In choosing the parameter along the radial direction, χ, into a coordinate system as (t, χ, θ, ϕ), we define the coordinate parameter χ as follows: with conditions that t, θ and ϕ are constant, the line element along the radial direction is given by $ds^2 = a^2 d\chi^2$, that is to say, χ is the "proper comoving radial distance". For convenience we transform the coordinate (t, r, θ, ϕ) to (t, χ, θ, ϕ) by $r = a\, f_k(\chi)$, where $f_k(\chi) = \sin\chi$, χ and $\sinh\chi$ for $k = 1$, 0 and -1 respectively:

$$ds^2 = -dt^2 + a(t)^2 \left[d\chi^2 + f_k(\chi)\,(d\theta^2 + \sin\theta d\phi^2) \right] . \tag{14}$$

In denoting $r_c(t_c) = a(t_c)\, f_k(\chi_c)$, this χ_c should be constant because χ is the comoving coordinate on the junction surface. The circumference of great circle, C_{gc}, is calculated in this metric to be

$$C_{gc} = \int_{\Sigma_t,\, \chi=\chi_c,\, \theta=\pi/2} \sqrt{g_{\phi\phi}}\, d\phi = 2\pi\, a\, f_k(\chi_c) \quad (= 2\pi r_c\,) . \tag{15}$$

Then we obtain the radially directional differential of the great circle in the metric (14), D_{univ}, to be

$$D_{univ} = \left. \frac{dC_{g.c}}{d(a\chi)} \right|_{\Sigma_t,\, \chi_c} = 2\pi \frac{df_k(\chi_c)}{d\chi_c} . \tag{16}$$

The metric of a spherically symmetric object can be given by

$$ds^2 = -A(T,R)\, dT^2 + B(T,R)\, dR^2 + R^2 (d\theta^2 + \sin^2\theta d\phi^2) , \tag{17}$$

where $A(T,R)$ and $B(T,R)$ are the arbitrary independent functions of T and R. Hereafter we assume that the radius of a spherical object is smaller than that of a cell, therefore the metric (17) is the exterior space time of the object. In the 2nd step of constructing the cell lattice universe, we should pay attention to that the coordinates (T,R) of spherically object are not necessarily identical to (t,r) or (t,χ) of homogeneous and isotropic universe. When the origins of coordinates of both metrics coincide, the radiuses of junction surface in both coordinate systems coincide. That is, we have $R(t_c) = r_c(t_c)$

on the junction surface. The time T of the metric (17) on the junction surface can be expressed by a function of t_c and r_c as $T_c = T(t_c, r_c)$, which should reflect the junction condition and the time evolution of the universe. The radially directional differential of the great circle in the metric (17) should be calculated on Σ_t. The circumference of the great circle, C_{gc}, is given by eq.(15). With denoting the vector, e^μ, at the junction surface and orthogonal to Σ_t as $e^\mu = (dT_c, dr_c, 0, 0)$ in the coordinate of metric (17), the vector, v^μ, orthogonal to the junction surface and parallel to Σ_t can be obtained to be $v^\mu = (-g^{TT}dr_c, g^{RR}dT_c, 0, 0)$. This gives us the difference along the radial direction as $d(a\chi)^2 = g_{\mu\nu}v^\mu v^\nu$, and the difference of the circumference as $dC_{gc} = 2\pi dr_c = 2\pi v^R$. For the averaged cosmological time, we have the relation $-dt_c^{\,2} = g_{\mu\nu}e^\mu e^\nu$. Then we can calculate the radially directional differential of the great circle in the metric (17), which we denote as D_{BH}, as follows:

$$D_{BH} = \frac{2\pi dr_c}{d(a\chi)} = 2\pi \frac{g^{RR}dT_c}{\sqrt{g_{\mu\nu}v^\mu v^\nu}} = 2\pi \sqrt{-\frac{g_{TT}}{g_{RR}}}\frac{dT_c}{dt_c} \tag{18}$$

The junction condition, $D_{univ} = D_{BH}$, gives us the equations [4]:

$$\begin{aligned} a(t_c) &= \frac{r_c(t_c)}{f_k(\chi_c)}, \\ \left(\frac{dr_c(t_c)}{dt_c}\right)^2 &= \alpha_k^{\,2} - \frac{1}{B(t_c, r_c)}, \end{aligned} \tag{19}$$

where $\alpha_k = \cos\chi_c$, 1 and $\cosh\chi_c$, for $k = +1$, 0 and -1, respectively.

For the cell lattice universe made by Schwarzschild black holes in the Einstein gravity, the averaged scale factor is given by the eqs.(19) with $B^{-1} = 1 - 2G_0M/R$, where G_0 is the ordinary Newton constant. It can be easily checked that this equation reproduces the same expansion law as for dust-dominated Friedmann universe in the Einstein gravity [4].

4 Time evolution of Black Holes in Expanding Universe

4.1 Black hole mass and cell lattice universe

As mentioned in the section 1, our interest in this paper is the effects of the expansion of the universe and a massless scalar field on black holes. We try to extract such the effects from the cell lattice universe which is assumed to be meaningful as an approximation of multi-black hole space time even in the Brans-Dicke gravity. This issue can be discussed by the eqs.(19) once the function $B(T, R)$ is specified.

To specify the form of $B(T, R)$, we should note here that the uniqueness theorem [7] of the black holes in Brans-Dicke gravity has already been established with the asymptotically flat condition, not in the expanding universe [8]. That is, the spherically symmetric non-rotating black hole in the asymptotically flat space time is of the Schwarzschild black hole even in the Brans-Dicke gravity. Further it is important for specifying $B(T, R)$ to remark the issue peculiar to the scalar-tensor gravity: *gravitational memory effect*. This issue is expressed as: when a black hole is formed in the expanding universe, does scalar field on the event horizon keep the value at the black hole formation time during cosmological evolution? If the scalar field keeps its original value, it is said that the black hole has the gravitational memory effect, if not, the black hole don't have the memory [9]. Concerning with this issue, the reference [10] gives the argument by perturbation of a stationary black hole that the gravitational memory effect does not occur in scalar-tensor gravities. That is, the scalar field on the event horizon seems to have time dependence along with cosmological evolution. Then we adopt a naively acceptable ansaz as follows: the spherically symmetric black hole composing the cell lattice universe in Brans-Dicke gravity is of the same form as Schwarzschild black hole except that its radius depends on the cosmological time, $R_g(t_c)$. With this ansaz we define a mass of the black hole in the cell lattice universe, M, in Brans-Dicke gravity as,

$$M(t_c) \equiv 8\pi\varphi(t_c)R_g(t_c)\,. \tag{20}$$

That is, we assume the black hole is of "Schwarzschild type" given by eq.(17) on the junction surface with

$$A(t_c, r) = B(t_c, r_c)^{-1} = 1 - \frac{M(t_c)}{8\pi\varphi(t_c)\, r_c}\,. \tag{21}$$

This mass, M, has certainly the dimension of mass, but it is not obvious whether this definition can be consistent with the ADM mass defined by the generator of Killing time translation in a stationary space time. Provided it is shown that the M changes adiabatically in cosmological time scale and that the black hole horizon size is much smaller than the cosmological one, we can consider that the black hole exists in a local asymptotically flat region as an ordinary Schwarzschild black hole and that the cosmological time t_c is equivalent to the time T of the metric (17) within the scale of black hole. In such the case our mass M can be effectively treated as the ADM mass defined by the generator of the local asymptotically time like Killing vector, where the Killing time is T of the matric (17).

With specifying $B(T, R)$ as above, the junction condition (19) gives

$$M(t_c) = 8\pi\beta_k^3\,\varphi(t_c)\,a(t_c)\left[\dot{a}(t_c)^2 + k\right]\,, \tag{22}$$

where $\beta_k = f_k(\chi_c)$. Here motivated by the comment at the last paragraph in the previous section, we assume the cell lattice universe is the well-defined averaged model of inhomogeneous universe reproducing the expansion law of Friedmann model even in the Brans-Dicke gravity. Because of this assumption, the averaged scale factor, $a(t_c)$, and the scalar field, $\varphi(t_c)$, are given by the dust-dominated Friedmann universe in Brans-Dicke gravity. The time evolution of $M(t_c)$ can be investigated with the eqs.(5), (8) and (22). It is not a priori obvious whether or not the mass M adiabatically depends on the cosmological time.

As is already pointed out in the reference [10], provided our mass M can be considered as the ADM mass in a local asymptotically flat region, we can discuss whether the mass in the Einstein frame changes with time. With attaching a tilde to the quantity in the Einstein frame, we find the relation $M/\widetilde{M} = d\widetilde{T}/dT$, because the ADM mass is given by the generator of the local asymptotic Killing vector, which is normalized at the boundary of the local asymptotically flat region and scales inversely with the time T. Since the Killing time translation is given by the line element along integral curves of the time like Killing vector, we obtain the relation at the boundary of the local asymptotically flat region: $-dT^2 = -\exp[-\sigma/(2\omega+3)]\,d\widetilde{T}^2$, where dT and $d\widetilde{T}$ express the Killing time translations in the Brans-Dicke and the Einstein frames respectively, and the frame transformation given by eq.(10) is used. Therefore we obtain the relation of the masses [10]

$$M = \exp\left[\frac{\sigma}{4\omega+6}\right]\,\widetilde{M} \propto \sqrt{\varphi}\,\widetilde{M}\,. \tag{23}$$

If $\widetilde{M}$ is constant, M is proportional to $\sqrt{\varphi}$. In the case that the M defined by ep.(22) can be treated as the ADM mass, by comparing the M with $\sqrt{\varphi}$, we can obtain a suggestion about the time dependence of $\widetilde{M}$ which is the black hole mass in expanding universe in the Einstein gravity with the presence of scalar field.

4.2 Analysis of the black hole mass

Hereafter we denote the averaged cosmological time t_c simply by t.

In flat case $k = 0$, it is easily found from eqs.(13) and (22) that the mass M is constant, and that the radius $R_g(t) \propto 1/\varphi(t)$ decreases decelerately while the averaged scale factor continues to expand. Therefore the mass can be effectively treated as ADM mass after the universe expands enough to be much larger than the black hole. Clearly $M \not\propto \sqrt{\varphi}$. This means that the mass in the Einstein frame has time dependence.

In two cases $k = \pm 1$, we calculate the mass $M(t)$ numerically with *Mathematica* from $t = 0.5$ to $t = 50000$ for open case $k = -1$, and to $t = 1000$ for closed $k = 1$. We attach the subscript i to the quantities evaluated at $t = 0.5$, and the subscript f at $t = 50000$ for $k = -1$ and at $t = 1000$ for $k = 1$. As mentioned at the end of the section 2, the curved Friedmann universe can be effectively treated as flat one in the early stage of time evolution. Therefore we set the integration constant t_0 of eq.(8) be zero, $t_0 = 0$. The initial values of the averaged scale factor a_i and scalar field φ_i are related through eq.(12). Our choices of the other parameters are as follows: the initial value of scale factor, $a_i = 10$. The Brans-Dicke parameter $\omega = 500$, which is the experimental lower bound of ω. Another integration constant $\epsilon_0 = 100$. The size of one cell in the cell lattice universe $\chi_c = 0.9747$ and 0.1993 for $k = -1$ and 1 respectively [6]. We regard the end time of calculation as our present time, because, as shown in the followings, the change rate of the mass M in time evolution is sufficiently small in comparison with the averaged Hubble parameter at the end time for both cases $k = \pm 1$, where we define the averaged Hubble parameter by $H = \dot{a}/a$ with the averaged scale factor a.

For the open case $k = -1$, the averaged Hubble time at present $t = 50000$ is $1/H_f = 53931$. The table 1 shows some values of the averaged cosmological red shift which is defined by $z(t) = a_f/a(t) - 1$ with the averaged scale factor a. The fig.2 is for the change rate of $M(t)$ normalized by the averaged Hubble parameter: $H_m(t) = (\dot{M}/M)/H$. The H_m is of negative and asymptotes to zero for sufficiently late stage of cosmological evolution $t > 10000$ where $z < 3.170$. The change rate, for example at $t = 2$, takes the value $H_m(2) = 0.0001469$. This indicates that the time evolution of M is less significant than that of the averaged scale factor at least for $t > 2$ $(z < 1840)$, where $H_m < O(10^{-4})$. That is, M is adiabatic in this epoch. The radius of black hole R_g decreases decelerately with time, from $R_{g\,i} = 5210$ to $R_{g\,f} = 5142$, while the averaged scale factor continues to expand for all time. Therefore the mass M can be effectively treated as ADM mass after the universe expands enough to be much larger than the black hole. The fig.3 is the plot of M with SqM which is defined by $SqM(t) = M_f\sqrt{\varphi(t)/\varphi_f}$. The square root of the averaged scalar field SqM at the initial time is $SqM_i = 609.4$, and increases decelerately for all time. The change rate of mass H_m becomes zero at $t = 14.92$. The mass M increases from $M_i = 613.6$ to $M(14.29) = 613.9$, and decreases decelerately after the time $t = 14.92$ where $z(14.92) = 17.25$. This fig.3 indicates the mass M does not coincides with SqM, and that the mass in the Einstein frame has time dependence.

For the closed case $k = 1$, the averaged Hubble time at present $t = 1000$ is $1/H_f = 1743$. The table 2 includes some values of the averaged cosmological red shift $z(t) = a_f/a(t) - 1$. The fig.4 shows H_m, the change rate of M normalized by the averaged Hubble parameter. The change rate H_m decreases

Time t	0.5	2	10	20	100	500	2000	5000	10000	30000
Red Shift z	5559	1840	601.8	375.8	125.5	40.42	14.06	6.376	3.170	0.5964

Table 1: Table of the averaged cosmological time and red shift for open case $k = -1$. The present time is $t = 50000$.

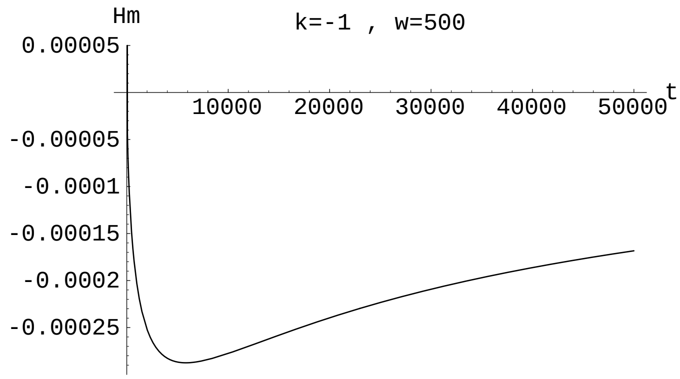

Figure 2: Graph of $H_m = (\dot{M}/M)/H$ for $k = -1$, the change rate of the mass. $H_{m\,i} = 1.00283$, $H_m(2) = 0.0001469$ and $H_m(14.92) = 0$.

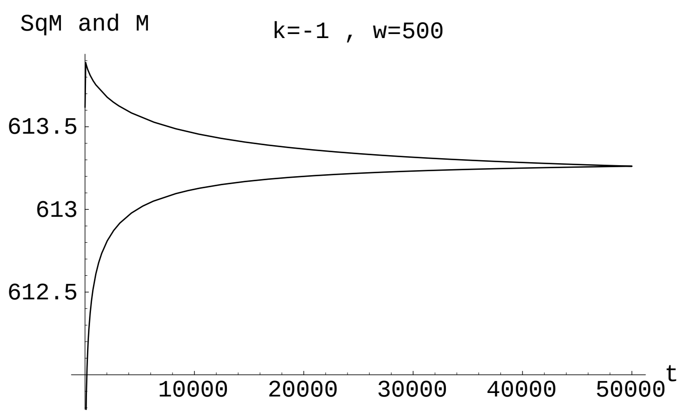

Figure 3: Graph of $M(t)$ and $SqM(t)$ for $k = -1$. The upper curve is for M while the bottom one for SqM. In this graph the overall scale of SqM is set so that $M_f = SqM_f$. $M_i = 613.6$, $M(14.29) = 613.9$ and $SqM_i = 609.4$.

Time t	0.5	2	5	10	15	20	50	100	300	500
Red Shift z	174.4	57.27	29.73	18.21	13.64	11.09	5.592	3.189	1.072	0.5085

Table 2: Table of the averaged cosmological time and red shift for closed case $k = 1$. The present time is $t = 1000$.

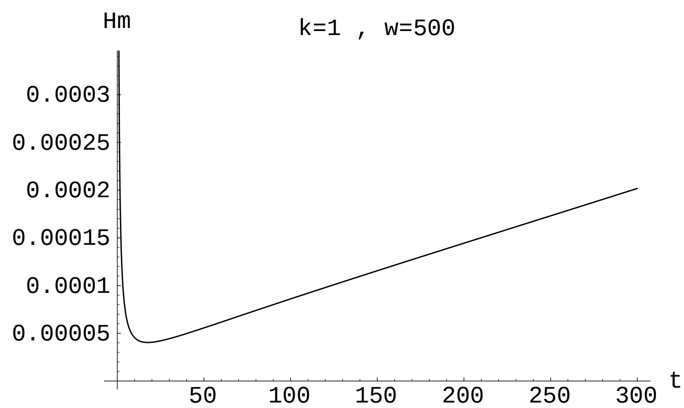

Figure 4: Graph of $H_m = (\dot{M}/M)/H$ for $k = 1$, the change rate of the mass. $H_{m\,i} = 1.00282$, $H_m(2) = 0.0001565$ and $H_{m\,f} = 0.0006698$.

rapidly in the early time, and turns to increase around $t = 20$ where $z = 11.09$. At present, the change rate is $H_{m\,f} = 0.0006698$. The increase of H_m continues up to the turning time of the averaged scale factor from expansion to contraction, where the turning time is $t = 5483$. The change rate, for example at $t = 2$, is $H_m(2) = 0.0001565$. This indicates that the time evolution of the mass M is adiabatic within the epoch from $t = 2$ to the present $t = 1000$, where $H_m < O(10^{-4})$. The black hole radius R_g decreases decelerately with time, from $R_{g\,i} = 27.54$ to $R_{g\,f} = 27.29$, while the averaged scale factor lasts to expand untill the turning time $t = 5483$. Therefore the mass M can be effectively treated as ADM mass after the universe expands much larger than the black hole. The fig.5 is for the plot of M and SqM. The square root of the averaged scalar field SqM increases decelerately for all time from the initial value $SqM_i = 3.227$. The mass M takes the initial value $M_i = 3.224$, and also increases decelerately for all time. This fig.5 indicates that M and SqM do not coincide with each other, and that the mass in the Einstein frame has time dependence.

5 Summary and Discussion

In order to investigate the effects of expansion of the universe and a scalar field on the celestial objects which compose the inhomogeneity of the universe, we constructed the cell lattice universe in Brans-Dicke gravity. The assumptions in constructing the cell lattice universe were that the black holes included in the universe are of the “Schwarzschild type”, eqs.(17) and (21), and that the averaged scale factor and scalar field are given by those of the Friedmann universe. In constructing the cell lattice universe we defined the mass M by eq.(20) which is equivalent to the ordinary ADM mass in the case of adiabatic time evolution of M. Further we obtained the eq.(22) to calculate the time evolution of the black hole

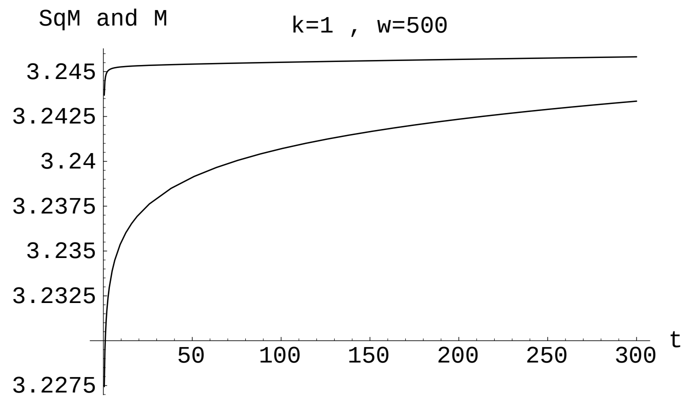

Figure 5: Graph of $M(t)$ and $SqM(t)$ for $k = 1$. The upper curve is for M while the bottom one for SqM. In this graph the overall scale of SqM is set so that $M_f = SqM_f$. $M_i = 3.244$ and $SqM_i = 3.227$.

mass M. It is not a priori known whether or not the mass depends on time.

As the results by analytical and numerical methods, the M behaves in qualitatively different way with respect to the value of k. The mass M decreases decelerately for open case $k = -1$, stays completely constant for flat case $k = 0$ and increases decelerately for closed case $k = 1$. The change rate of the mass H_m is very small for both cases of $k = \pm 1$, $H_m < O(10^{-4})$ with our numerical results. This means that the M evolves in time adiabatically. Because here we assume that the averaged scale factor in eq.(22) is given by that of the Friedmann model in Brans-Dicke gravity, the adiabaticity of M is consistent with the fact that the cell lattice universe constructed with the Schwarzschild black hole whose mass is completely constant reproduces the expansion law of the Friedmann universe in Einstein gravity. Further the radius R_g decreases decelerately for every case of $k = \pm 1$, 0, while the averaged scale factor continues to increase. This behavior of R_g can be easily understood for the case $k = 0$, which is $R_g \propto t^{-2/(3\omega+4)}$. Therefore it is reasonable to consider the black hole is in a local asymptotically flat region after the universe expands enough to be much larger than the black hole, consequently the mass M is equivalent to the ADM mass defined by the local asymptotic Killing time translation.

Further we can recognize by the eq.(23) that the black hole mass in Einstein frame has time dependence for every case of $k = \pm 1$, 0. According to the uniqueness theorem [7] [8], non-rotating and non-charged black hole in asymptotically flat space time is specified only by the ADM mass which is completely constant in this theorem. The time dependence of the mass may indicate that the uniqueness theorem in an expanding universe will be broken in the cosmological time scale. It is a very interesting problem remained to be solved.

In the numerical calculations shown in the section 4, we set $\omega = 500$ which is the experimental lowest value. If the parameter is set as $\omega < 500$, the absolute value of the change rate of the mass M tends to increase. However the behavior of the mass M and the radius R_g for the case $\omega < 500$ are the same as for the case $\omega = 500$, consequently the mass can be equivalent to the ADM mass once the universe expands enough to be much larger than the black hole size.

So far we have treated only the Brans-Dicke gravity as a representative case of scalar-tensor gravities in which the parameter ω depends on the scalar field, and have not introduced any potential of the scalar field. Even if the averaged scale factor and scalar field in the cell lattice universe in a general scalar-tensor gravity or with a potential of the scalar field behave quite differently from those in the

Brans-Dicke gravity in the early universe, in the case they asymptote to those in the Brans-Dicke gravity at least in a sufficiently late stage of the expansion of the universe, the same arguments and results mentioned until the last paragraph are true of this case. Further in the case that the averaged scale factor or scalar field in a general theory of gravity always behaves differently from that in the Brans-Dicke gravity, though we need to reanalyze the eq.(22) in order to know the details of the time dependence of M, however it is natural to propose that the mass M depends on time in this case.

Let us remark about our assumption, the "Schwarzschild type" ansaz of black hole composing the cell lattice universe given by the eqs.(17), (20) and (21). The construction of the cell lattice universe that a cell is replaced by a black hole, means that the dust-matter on homogeneous and isotropic universe in a cell is concentrated at a center point of the cell. Therefore it is appropriate to consider the black hole space time replacing the cell, includes only the scalar field coupling with gravity in Brans-Dicke frame. The field equation of such the scalar field is obtained by substituting $T_{\mu\nu} = 0$ into the eqs.(2),

$$G_{\mu\nu} = \frac{\omega}{\varphi^2}\left[(\nabla_\mu\varphi)(\nabla_\nu\varphi) - \frac{1}{2}g_{\mu\nu}(\nabla\varphi)^2\right] + \frac{1}{\varphi}\nabla_\mu\nabla_\nu\varphi\,, \tag{24}$$

where the non-zero components of the Einstein tensor are calculated from the eqs.(17), (20) and (21),

$$\begin{aligned} G_{01} &= \frac{\dot{R}_g}{R^2 - R\cdot R_g}\,, \\ G_{22} &= -R^3\left[\frac{\ddot{R}_g}{2(R-R_g)^2} + \frac{\dot{R}_g{}^2}{(R-R_g)^3}\right]\,, \\ G_{33} &= \sin^2\theta\, G_{22}\,, \end{aligned} \tag{25}$$

where $\dot{R}_g = dR_g/dT$ with the time T of metric eq.(17). The scalar field on the Schwarzschild type space time should satisfy the above field eqs.(24) and (25).

Our ansaz of the Schwarzschild type black hole is motivated by the uniqueness theorem of the black hole in asymptotically flat space time [8] and the indication in the reference [10] that the gravitational memory effect seems not to occur. Reversely, in paying attention to the latter motivation, the gravitational memory effect can be discussed by the eqs.(24) and (25) with the boundary condition: $\varphi(T)$ at $R = r_c(T)$ is given by the scalar field of Friedmann model, where $r_c(T)$ is determined by the junction of our cell lattice universe in the section 3. By investigating this system we can know whether the scalar field φ has spatial dependence or not. If not, it means that the gravitational memory effect does not occur in Brans-Dicke gravity, and that the indication in the reference [10] and our discussion in this paper are supported. The model of space time in such an approach to the gravitational memory effect, can be considered as a modified *swiss cheese universe* [11]. In the ordinary swiss cheese model, a spherically symmetric region in Friedmann universe is replaced by an ordinary Shcwarzschild black hole of constant radius and mass, but here we replace the spherical region by the "Schwarzschild type" black hole. This is the interesting and solvable problem.

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